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INPUT EFFICIENCY PROFILING: AN APPLICATION TO AIRLINES

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Scope and Purpose—An obvious measure of efficiency or productivity is the ratio of output to input. When there are multiple outputs and/or inputs one approach is to take weighted combinations before calculating the ratio. How should these weights be chosen? In comparing the efficiency of organizational units data envelopment analysis (DEA) allows each unit to employ its own weights so as to maximise its score subject to the condition that these weights do not cause any unit to attain a score exceeding 100%. One difficulty that arises with DEA is that when the number of units being compared is small there is poor discrimination and a large proportion of them are rated as 100% efficient. Also the DEA assumption that all inputs at a given ('naturally enveloped' or dominated) firm will be inefficient to the same degree seems to be unrealistic and unnecessary. This article presents a more searching and discriminating way of showing managers the source and extent of inefficiencies by simply applying DEA to each input resource separately in the case when inputs cannot be substituted for each other. If two or more inputs are substitutes then these are analysed together. By way of illustration the approach is used to compare major passenger airlines.

Abstract—Data envelopment analysis (DEA) can produce results which lack discrimination, one consequence of this is that a large proportion of decision-making units (DMUs) appear to be efficient. In addition, because it is a radial measure of efficiency it assumes that all inputs at a naturally enveloped production unit need to be reduced by the same proportion for efficiency to be achieved. It would seem to be more realistic to expect different inputs to have different efficiencies associated with them. A method is presented which retains the original spirit of DEA in trying to extract as much information as possible from the data without applying value judgments in the form of additional constraints. We propose that inputs which are not substitutes for each other be assessed separately and only with respect to outputs which consume them or to which they are otherwise related. In this way input-specific efficiency ratings are derived giving a profile for each DMU. When applied to a data set of 14 airlines the method uncovers inefficiencies which DEA could not find. DEA found half of the airlines to be fully efficient in all factors, whereas our profiling approach was more discriminating and showed that none of the airlines were efficient in all three of the inputs considered. This highlights a significant difference with DEA: by investigating the utilisation of individual inputs we are able to identify best-practice in each area. It is quite possible that no unit demonstrates best-practice in every area and so each unit will have targets to work towards—this is intuitively appealing as well as providing a link with the philosophy of best practice benchmarking.

INTRODUCTION

For analysts who were used to a multiplicity of simple ratio measures involving a single output and a single input, the appearance of data envelopment analysis (DEA) must have seemed like the answer to many of their problems, for it appeared to objectively combine all factors in a single measure. In its ratio form a DEA efficiency score is a sum of weighted outputs divided by a sum of weighted inputs, with individually calculated optimal weights for each firm or decision-making unit (DMU). However such analysts may have been disappointed if the number of DMUs being compared was small, for in such cases a large proportion of them will be rated as 100% efficient. This problem becomes more serious if the number of inputs or outputs is increased—i.e. more data actually makes things worse. This lack of discrimination is because of the great freedom in choosing the weights (also called multipliers): each factor adds another dimension to the feasible region—greatly enhancing the possibility of finding weights which will make units appear efficient. If there are few units to be compared then the constraints which limit the set of solutions will also be few in number.

As this difficulty is fairly well known it has been addressed in a number of ways. There is much in the literature that deals with placing various types of restrictions on the weights in order to obtain more acceptable results. Such methods normally involve either value judgments or arbitrary *a priori* bounds. We shall not review these here but Allen *et al.* [1] provide a survey. This article embarks upon a different

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direction—that of extracting more information from the data without making value judgments. We aim to compare the efficiency with which each resource input is being utilised by each DMU to generate outputs. We do this by applying a DEA-type analysis to each input in turn and only including those outputs to which the given input is related. This produces a set of input efficiency scores for each unit, which we refer to as a profile. In what follows we shall use the term 'conventional DEA' to represent the form which treats all inputs together.

The efficiency score of an operational unit obviously depends on the position and shape of the efficient frontier from which it is derived. This frontier determines what is theoretically achievable. In parametric estimations (used in econometrics) one has a variety of production functions with which to model the frontier e.g. Cobb-Douglas, translog, CES, generalised Leontief etc.; each of these models has its own underlying assumptions. The application of such approaches to deal with each input separately has been carried out by Kopp [2] and Kumbhakar [3]. The latter author felt that "knowing the magnitude of [overall] technical efficiency is not enough. It is important to know which inputs are causing the inefficiency and to what extent", and so any measure requiring an equiproportional reduction in all inputs to achieve efficiency was too restrictive. Kumbhakar's investigation [3] using panel data relating to U.S. railroads found that inefficiency because of labour was of a much higher degree than that caused by fuel, this was corroborated by a historical review of labour practices in the railroad industry.

Turning now to DEA, this is described as a non-parametric approach because it does not assume a particular functional form for the entire frontier. However it does make an assumption regarding the shape of the individual segments or facets that are used to model the efficient frontier; models that have been used include: piecewise linear (the most widely known), piecewise Cobb-Douglas, and piecewise log-linear (these are all described in Chapter 2 of Charnes *et al.* [4]). In DEA equiproportional reduction of all inputs (or expansion of all outputs) to achieve efficiency is an assumption in the models most frequently used—in fact it is a consequence of using a radial measure that the 'input mix' is kept the same. A radial efficiency measure uses a line in input space from the origin O to the point P being analysed. If P is not efficient then this line will cross the frontier at F and the ratio OF/OP gives the efficiency score. Ideally F will lie on a line or facet between two or more efficient units, P is then said to be 'naturally enveloped' by these 'reference units'; the point F will then provide the target values for unit P. However this natural envelopment will not always occur and the frontier then has to be artificially extended parallel to one of the coordinate axes. This gives rise to input slacks which are additional reductions in particular inputs beyond those arising from equiproportionate reduction. These additional improvements are a consequence of incomplete frontiers and not because DEA has cleverly identified variations in input-specific efficiency. A disadvantage associated with slacks is that they are not reflected in the efficiency score as this score only measures the radial contraction. Input slacks cannot occur if only one input is being analysed at a time and so that problem is removed, however output slacks are still possible.

In summary, parametric approaches have been applied to each input individually and we aim to do the same for the non-parametric approach. The key advantage is the removal of the assumption that inefficiency occurs to the same degree in every input, thus for inefficient units we will no longer be required to reduce all inputs by the same proportion to achieve efficiency. The remainder of this paper is arranged as follows: the proposed technique is described in the next section, there then follows an application to data on fourteen major international passenger airlines. The results are then compared with those of conventional DEA.

INPUT EFFICIENCY PROFILING

Suppose that resource x_i acts as an input to s outputs y_r $(r=1,\dots,s)$; we emphasise that this may be a *subset* of all t outputs $(s \leq t)$, for example a lecturer's time (input) spent teaching undergraduates does not contribute to research output, similarly the number of pigs a farmer rears does not contribute to his farm's milk production. For this reason we do not treat such unrelated variables together. Note that a different resource may act as an input to a different set of outputs, possibly fewer or more. The relative efficiency (E_{ik}) with which resource i is being utilised to produce the relevant outputs by DMU k is evaluated using the following L.P.(linear program) in which the u-variables are being solved for, and the y's and x's are the observed output and input values respectively.

$$\text{Maximize } E_{ik} = \frac{\sum_{r=1}^{s} u_{irk} y_{rk}}{x_{ik}} \tag{1}$$

subject to

$$\frac{\sum_{r=1}^{s} u_{irk} y_{rj}}{x_{ij}} \leq 1, j=1,...,n \tag{2}$$

and

$$u_{irk} \geq \epsilon, \ r=1,...,s \tag{3}$$

Where ϵ is a small positive number, n is the number of DMUs and u_{irk} is the weight attached to output r when evaluating the efficiency of input i of DMU k. As with DEA each DMU has its own set of weights. The key difference between this and the DEA formulation is that here each linear program only deals with a single input rather than a weighted sum of all inputs. Thus instead of a single efficiency score we now have a score for each resource input. Unlike conventional DEA there are no longer any weights on the input variables, thus we no longer have to impose additional conditions on the weights *a priori* in order to avoid the possibility of placing a zero/epsilon or any other unrealistic weight on any input. It is no longer possible to hide poor/wasteful utilisation of any resource. Dyson and Thanassoulis [5] have rightly pointed out that in basic DEA models "the worst performance aspects are all but ignored in the assessment".

We now show how our input-specific efficiency scores may be interpreted. Consider a branch which uses two inputs and has a score of 0.8 in relation to input 1 and 0.9 for input 2, this means it could aim to maintain the same outputs as before but using only 80% of the current level of input 1 and 90% of the current level of input 2. If instead we are interested in raising output levels, then based on input 1 the outputs are currently 20% below the projected value, and 10% below in relation to input 2. If we choose the lower output target (that associated with the 10% figure) then we should keep input 2 fixed at its current level whilst also achieving a reduction in input 1. Whereas if we select the higher output target we shall fully utilise input 1 and we shall need additional supplies of input 2.

It may be worth noting that the objective function in the above LP is in fact a linear combination of simple ratios each with the same input denominator. Hence profiling in the present context may be viewed as lying somewhere between ratio analysis and DEA.

Note that a production unit cannot appear to be efficiently utilising a small quantity of one input by using a large amount of another input since, by hypothesis, the inputs are non-substitutable. If, however, in a given situation two or more inputs are substitutes then they should be dealt with together in the same LP. This is achieved by placing a linear combination of the inputs in the denominator of (1) and (2), the weights being determined by the optimisation model. For instance if x_2 and x_3 were substitutes then to evaluate the efficiency with which these were being utilised the denominator of (1) and (2) would be replaced by $v_{1k}\, x_{1k}+v_{2k}\, x_{2k}$, where the v's are weight variables to be obtained by the optimization process. (Note that the resulting problem can still be solved as a linear programme by including the normalising constraint $v_{1k}\, x_{1k}+v_{2k}\, x_{2k}=1$.) Since the weights are interpreted as trade-offs between inputs (Chang and Guh [6], Norman and Stoker [7], p. 47) or marginal rates of substitution, it follows that in DEA there is an implicit assumption that any input can act as a substitute for any other since it employs a weighted combination of all the inputs. However this is clearly not always appropriate: for example in a power plant the fuel and the staff employed cannot be substituted for each other, and in the case of a university the professors and secretaries cannot normally replace each other. It then becomes unclear what physical interpretation the weights arising from conventional DEA can have.

APPLICATION TO MAJOR AIRLINES

The data to be used (Table 1) has been taken from Schefczyk [8] and covers 14 major international passenger carriers for the year 1990, (we have excluded one of the airlines because it only transported cargo). The variables are as follows:

x_1 = aircraft capacity in ton kilometres
x_2 = operating cost
x_3 = non-flight assets (all assets not already reflected in x_1) e.g. reservation systems, facilities, current assets)
y_1 = passenger kilometres
y_2 = non-passenger revenue

Table 1. Input and output variable values

Airline	Inputs: Available ton km	Inputs: Operating cost	Inputs: Nonflight assets	Outputs: Revenue pass. km	Outputs: Nonpass. revenue
Air Canada	5723	3239	2003	26677	697
All Nippon Airways Co.	5895	4225	4557	3081	539
American Airlines, Inc.	24099	9560	6267	124055	1266
British Airways Plc	13565	7499	3213	64734	1563
Cathay Pacific Airways	5183	1880	783	23604	513
Delta Air Lines, Inc.	19080	8032	3272	95011	572
IBERIA Lineas Aereas	4603	3457	2360	22112	969
Japan Airlines	12097	6779	6474	52363	2001
KLM Royal Dutch Airlines	6587	3341	3581	26504	1297
Korean Air	5654	1878	1916	19277	972
Lufthansa	12559	8098	3310	41925	3398
Quantas	5728	2481	2254	27754	982
Singapore Airlines	4715	1792	2485	31332	543
UAL Corporation	22793	9874	4145	122528	1404

We presume that each input is related to both outputs when generating the profiles, though such an assumption does not in general have to be made with the profiling method. Table 2 displays as percentages both the DEA efficiency scores according to the Charnes, Cooper, Rhodes input minimisation model, constant returns to scale formulation (see [4] Chapter 2), as well as the input efficiency profiles.

The first thing to notice is that according to DEA seven of the fourteen airlines are deemed efficient (score of 100% and no slacks; slack values are not displayed here). This illustrates the lack of discrimination mentioned in the introduction. So according to DEA half of the airlines in our data set do not need to make any adjustments whatsoever in their input or output levels. Anyone using these results as a launching point for best-practice benchmarking of these seven airlines would therefore not be able to make any recommendations nor set any targets for the future. Much greater discrimination is shown by the profile scores (also using a constant returns to scale formulation). One observes that none of the airlines are efficient in all three inputs; this is perhaps more in keeping with what one would expect: that there will usually be some area where an improvement is due. Singapore Airlines is efficient in two inputs but is only 42% efficient in its utilisation of non-flight assets. Quantas and UAL have a DEA score of 100% and yet do not obtain this rating in any of the three input efficiencies. In fact the DEA score must always be at least as large as the largest of the input efficiencies which form our profile. This is because the latter are special cases of DEA with zero weights on all but one of the inputs i.e. they are just some of the feasible solutions in the DEA LP: the feasible regions in the profiling LPs for a given unit are subsets of the feasible region in the DEA LP for that unit.

We can use conventional DEA to provide efficiency ratings for individual inputs by radially projecting onto the frontier and then adjusting for any slack in the given input. Taking this as a target value, the conventional DEA input efficiency is then the ratio of the target input to the observed input. These are displayed as $E(x_1)$ etc. in Table 2 for comparison purposes. We see that the reductions that can be made

Table 2. Percentage efficiency scores according to DEA and input profiling

	Method									
	DEA Constant returns to scale model				Profiling Constant returns			Profiling Variable returns		
Airline	Overall	$E(x_1)$	$E(x_2)$	$E(x_3)$	E_1	E_2	E_3	E_1	E_2	E_3
Air Canada	87	87	72	87	79	56	49	81	56	50
All Nippon	84	84	45	58	84	45	24	86	46	24
American Airlines	95	95	95	95	77	74	66	100	100	100
British Airways	96	96	83	96	79	57	71	94	81	83
Cathay Pacific	100	100	100	100	73	79	100	89	95	100
Delta Air	98	98	95	98	75	68	96	91	89	98
Iberia	100	100	100	100	100	57	47	100	59	50
Japan Airlines	86	86	86	69	85	64	37	95	90	40
KLM	95	95	95	61	89	76	40	89	89	41
Korean Air	100	100	100	100	77	100	56	82	100	62
Lufthansa	100	100	100	100	100	81	100	100	100	100
Quantas	100	100	100	100	92	89	54	93	98	54
Singapore Airlines	100	100	100	100	100	100	42	100	100	42
UAL	100	100	100	100	81	71	98	100	100	100

in any given input (i.e. potential improvements) are in every instance greater according to profiling than according to DEA. The overall DEA score is in fact the largest of the individual ratios for a given unit; one can reduce all inputs to this proportion and still maintain current output levels. Using this line of argument we might select the largest input efficiency from the profile of each DMU as being the overall efficiency score. These will of course be lower than the overall scores from DEA and so our approach will be more demanding in terms of the degree of suggested improvement at each DMU. This is consistent with the oft-stated line that conventional DEA shows each unit in the best possible light. Its single figure scores are obtained by emphasising strengths and downplaying weaknesses. By contrast, input profiling tries to shine the light in corners that some might prefer to remain in darkness.

The above analysis assumed constant returns to scale which, according to Good *et al.* [9] is consistent with the vast majority of the airline literature. Nevertheless we have also carried out our analysis allowing for variable returns to scale (Banker, Charnes Cooper input minimisation model [10]) and present in the last three columns of Table 2 the associated results. As one might expect, the greater flexibility of variable returns leads to scores which are at least as good and often higher. We have that three airlines (American, Lufthansa and UAL) now have perfect profiles whereas none did previously. Interestingly American Airlines and UAL are the two largest in our data set, so there is some evidence here of decreasing returns to scale. When conventional DEA with variable returns is applied (results not shown in table) we find one more airline (American) appearing fully efficient, making eight in total; the remaining units show small increases in score apart from Japan Airlines which jumps from 86% to 95%. Clearly, what is theoretically achievable depends on the model being used—different models lead to different frontiers. One would also expect different results when comparing traditional parametric production function models with those which are input-specific. The approach we have taken here is consistent with that which underlies the benchmarking philosophy: to look for the best in all spheres—to identify areas of best-practice in other firms and attempt to combine them all together in one firm by the setting of appropriate targets.

SUMMARY

DEA is a radial measure of efficiency and so assumes that an inefficient unit which is naturally enveloped by its reference set must contract all its inputs in the same proportion to become efficient. We take the view that in any organization it is more likely that some inputs will be utilised less efficiently than others and that it would be useful to managers to identify these. Another disadvantage which sometimes arises with conventional DEA is that a large proportion of the operational units being compared turn out to be 100% efficient. Hence we have the possibility of poor discrimination in two different ways: firstly in identifying the inefficient units and secondly in identifying which inputs used by those units give the greatest cause for concern. This article has attempted to tackle these problems not by adding constraints based on either a priori or subjective judgments, but by taking each resource input in turn and only analysing it together with those outputs it affects or which consume it. Such an approach is appropriate if the inputs are not substitutes for each other. If two or more inputs are substitutable then they must be analysed together. DEA uses such a linear combination of all the inputs in its formulation. In this light we now see that DEA is in fact the special case where every input can act as a substitute for every other and each input is consumed by every output.

By taking each input separately there is no longer any scope for extreme or unrealistic weights on the inputs since they are not weighted at all. This significantly cuts down the dimensions and hence the size of the solution space by comparison with DEA. As a result the computed scores no longer show a large proportion of units displaying 100% ratings. For DEA-inefficient units too the profiles show more scope for improvement. This improved discrimination was demonstrated on data for fourteen major air carriers.

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