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# A neutral DEA model for cross-efficiency evaluation and its extension ☆

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ABSTRACT

Cross-efficiency evaluation has long been suggested as an alternative method for ranking decision making units (DMUs) in data envelopment analysis (DEA). This paper proposes a neutral DEA model for cross-efficiency evaluation. Unlike the aggressive and benevolent formulations in cross-efficiency evaluation, the neutral DEA model determines one set of input and output weights for each DMU from its own point of view without being aggressive or benevolent to the other DMUs. As a result, the cross-efficiencies computed in this way are more neutral, neither aggressive nor benevolent. The neutral DEA model is then extended to a cross-weight evaluation, which seeks a common set of weights for all the DMUs. Numerical examples are provided to illustrate the applications of the neutral DEA model and the cross-weight evaluation in DEA ranking.



## 1. Introduction

Data envelopment analysis (DEA) developed by Charnes, Cooper, and Rhodes (1978) has been widely accepted as a powerful performance assessment tool, which provides each decision making unit (DMU), a not-for-profit entity or organization that consumes multiple inputs to produce multiple outputs, with a good opportunity to self-evaluate its efficiency relative to the other DMUs. The self-evaluated efficiencies are then compared and ranked. Since the self-evaluation allows each DMU to rate its efficiency with the most favorable weights to itself, more than one DMU is often evaluated as DEA efficient and cannot be discriminated any further. So, lack of discrimination power is one of the major drawbacks that DEA suffers from. It also causes another significant problem that the self-evaluation allows each DMU to be evaluated with its most favorable weights. That is, the inputs and outputs favorable to a particular DMU will be heavily weighted, whereas those not favorable to the DMU will be less weighted or ignored. So, the weights determined by the self-evaluation may sometimes not be realistic.

To increase the discrimination power of DEA and make its weights more realistic, cross-efficiency evaluation has been suggested as an alternative method to the self-evaluation and an extension to DEA. The cross-efficiency evaluation requests each DMU not only to be self-evaluated but also to be peer-evaluated. The concept of cross-efficiency evaluation was first proposed by Sexton, Silkman, and Hogan (1986) and was later examined in detail by Doyle and Green (1994, 1995). In the cross-efficiency evaluation, each DMU determines a set of weights that are either aggressive or benevolent to the others, leading to $n$ sets of weights available for $n$ DMUs. Each DMU is then evaluated with the $n$ sets of weights, respectively, leading to $n$ efficiency values. The $n$ efficiency values for each DMU are finally averaged as an overall efficiency value for the DMU. It is believed that the cross-efficiency evaluation can guarantee a unique ordering for the DMUs and can be used with few DMUs (e.g. four or five) to produce a unique ordering (Doyle & Green, 1995).

Due to its power in discriminating among DMUs, the cross-efficiency evaluation has found a significant number of applications in the DEA literature. For example, Oral, Kettani, and Lang (1991) used the cross-efficiency evaluation for R&D project selection. Shang and Sueyoshi (1995) utilized the cross-efficiency evaluation to select the most efficient flexible manufacturing systems (FMSs). Green, Doyle, and Cook (1996) employed the cross-efficiency evaluation for preference voting and project ranking. Baker and Talluri (1997) applied the cross-efficiency evaluation for industrial robot selection. Talluri and Sarkis (1997) illustrated the use of cross-efficiency for evaluating cellular layouts. Sun (2002) used the cross-efficiency evaluation to differentiate between good and bad computer numerical control (CNC) machines. Chen (2002) used the cross-efficiency evaluation to identify the overall efficient and 'false standard' efficient electricity distribution sectors in Taiwan. Ertay and Ruan (2005) utilized the cross-efficiency evaluation to determine the best labor assignment in cellular manufacturing system (CMS). Lu and Lo (2007a, 2007b) examined the economic-environmental performances of 31 regions in China by taking into

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account various environmental factors and integrated the cross-efficiency evaluation with cluster analysis to construct a benchmark-learning roadmap for those inefficient regions to improve their efficiencies progressively. Wu, Liang, Wu, and Yang (2008) and Wu, Liang, and Yang (2009a) applied the cross-efficiency evaluation in conjunction with cluster analysis for Olympic ranking and benchmarking.

Apart from the applications mentioned above, theoretical research has also been conducted on the cross-efficiency evaluation. Anderson, Hollingsworth, and Inman (2002) proved in the case of single input and multiple outputs the fixed weighting nature of the cross-efficiency evaluation. They demonstrated with a numerical example how this unseen fixed set of weights might still be unrealistic. Sun and Lu (2005) presented a cross-efficiency profiling (CEP) model which was based upon a combination of the profiling approach developed by Tofallis (1996, 1997) and the cross-efficiency measure presented by Doyle and Green (1994). The CEP model evaluates each input separately and only with respect to the outputs that consume the input. In this way, input-specific ratings based on the cross-efficiency evaluation were derived to give a profile for each DMU. Bao, Chen, and Chang (2008) offered an alternative interpretation to the cross-efficiency evaluation from the viewpoint of slack analysis in DEA. Liang, Wu, Cook, and Zhu (2008b) extended the cross-efficiency model of Doyle and Green (1994) by introducing a number of alternative secondary goals for the cross-efficiency evaluation. Liang, Wu, Cook, and Zhu (2008a) generalized the cross-efficiency concept to game cross-efficiency by viewing each DMU as a player that seeks to maximize its own efficiency under the condition that the cross-efficiency of each of the other DMUs does not deteriorate and the cross-efficiencies as payoffs. A convergent iterative algorithm was presented to derive the best average game cross-efficiency scores, which constitute a Nash equilibrium point. Wu, Liang, and Chen (2009) extended the game cross-efficiency model of Liang et al. (2008a) to variable returns to scale (VRS) and presented a modified DEA game cross-efficiency model by appending an extra constraint to avoid producing negative cross-efficiencies under VRS. The model was then applied to Olympic rankings. Wu (2009) presented a revised benevolent cross-efficiency model and used the cross-efficiencies obtained to construct a fuzzy preference relation instead of the use of average cross-efficiency for ranking DMUs. Wu, Liang, and Yang (2009b) examined the cross-efficiency evaluation from the viewpoint of cooperative game and computed ultimate cross-efficiency by weighting $n$ cross-efficiency values rather than simply averaging them, where the weights for ultimate cross-efficiency were determined by using the Shapley value in cooperative game. Wu, Liang, Yang, and Yan (2009) and Wu, Liang, Zha, and Yang (2009) also developed a bargaining game model and a mixed integer programming model for the cross-efficiency evaluation. In the bargaining game model, each DMU is seen as an independent player and the bargaining solution between the CCR-efficiency and the cross-efficiency are obtained by using the classical Nash bargaining game model. It was shown that the bargaining efficiency was a Pareto solution. The mixed integer programming model was developed to find a best ranking order for each DMU.

From the literature review above, it is found that existing cross-efficiencies except for DEA game cross-efficiency were all computed either aggressively or benevolently. As a matter of fact, there is no guarantee that the two different formulations, aggressive and benevolent, can lead to the same efficiency ranking or decision conclusion. Although most of the applications utilized the aggressive formulation for cross-efficiency evaluation, there is no theoretical evidence to support such a choice. In particular, there have been no attempts so far to test if the two different formulations give the same ranking or conclusion. In this paper, we propose a neutral DEA model for cross-efficiency evaluation to avoid the difficulty in making a choice between the two different formulations. The neutral DEA model determines one set of input and output weights for each DMU without being aggressive or benevolent to the others. As a result, the cross-efficiencies will be more neutral. We then extend the neutral DEA model to determine a common set of weights for all the DMUs, which we refer to as cross-weight evaluation. We also provide a comparison between the cross-efficiency evaluation and the game cross-efficiency evaluation.

The rest of the paper is organized as follows. Section 2 describes the cross-efficiency evaluation and its aggressive and benevolent formulations. The neutral DEA model for cross-efficiency evaluation is developed in Section 3 and extended in Section 4. Comparisons of the cross-efficiency evaluation with the game cross-efficiency evaluation are provided in Section 5. Numerical examples are demonstrated in Section 6. Conclusions are offered in Section 7.

## 2. The cross-efficiency evaluation

Suppose there are $n$ DMUs to be evaluated against $m$ inputs and $s$ outputs. Denote by $x_{ij}$ $(i = 1, \ldots, m)$ and $y_{rj}$ $(r = 1, \ldots, s)$ the input and output values of $\mathrm{DMU}_j$ $(j = 1, \ldots, n)$, whose efficiencies are defined as

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \quad j = 1, \ldots, n, \tag{1}$$

where $v_i$ $(i = 1, \ldots, m)$ and $u_r$ $(r = 1, \ldots, s)$ are input and output weights.

Consider a DMU, say, $\mathrm{DMU}_k$, $k \in \{1, \ldots, n\}$, whose efficiency relative to the other DMUs can be measured by the following CCR model (Charnes et al., 1978):

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \\ \text{Subject to} \quad & \theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \leqslant 1, \quad j = 1, \ldots, n, \\ & u_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\ & v_{ik} \geqslant 0, \quad i = 1, \ldots, m, \end{aligned} \tag{2}$$

which aims to find a set of input and output weights that is most favorable to $\mathrm{DMU}_k$.

By using Charnes and Cooper transformation (Charnes & Cooper, 1962), model (2) can be equivalently transformed into the linear program (LP) below for solution:

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\ \text{Subject to} \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leqslant 0, \quad j = 1, \ldots, n, \\ & u_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\ & v_{ik} \geqslant 0, \quad i = 1, \ldots, m. \end{aligned} \tag{3}$$

Let $u_{rk}^*$ $(r = 1, \ldots, s)$ and $v_{ik}^*$ $(i = 1, \ldots, m)$ be the optimal solution to the above model. Then, $\theta_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ is referred to as the CCR-efficiency of $\mathrm{DMU}_k$, which is the best relative efficiency that $\mathrm{DMU}_k$ can achieve and reflects the self-evaluated efficiency of $\mathrm{DMU}_k$. As such, $\theta_{jk} = \sum_{r=1}^{s} u_{rk}^* y_{rj} / \sum_{i=1}^{m} v_{ik}^* x_{ik}$ is referred to as a cross-efficiency value of $\mathrm{DMU}_j$ and reflects the peer evaluation of $\mathrm{DMU}_k$ to $\mathrm{DMU}_j$ $(j = 1, \ldots, n;\ j \neq k)$.

Model (3) is solved $n$ times, each time for one different DMU. As a result, there will be $n$ sets of input and output weights available for $n$ DMUs and each DMU will have $(n-1)$ cross-efficiency values plus one CCR-efficiency value, which form a cross-efficiency value matrix, as shown in Table 1, where $\theta_{kk}$ $(k = 1, \ldots, n)$ are the CCR-efficiencies of the $n$ DMUs, i.e. $\theta_{kk} = \theta_{kk}^*$.

It is noticed that model (3) may have multiple optimal solutions. This non-uniqueness of input and output weights would damage the use of cross-efficiency evaluation if it were not resolved. To resolve this problem, one remedy suggested by Sexton et al. (1986) is to introduce a secondary goal which optimizes the input and output weights while keeping unchanged the CCR-efficiency determined by model (3). The most commonly used secondary goals were suggested by Doyle and Green (1994) and are shown below:

$$\begin{aligned}
\text{Minimize} \quad & \sum_{r=1}^{s} u_{rk}\left(\sum_{j=1, j\neq k}^{n} y_{rj}\right) \\
\text{Subject to} \quad & \sum_{i=1}^{m} v_{ik}\left(\sum_{j=1, j\neq k}^{n} x_{ij}\right) = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^{m} v_i x_{ik} = 0, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leqslant 0, \quad j = 1, \ldots, n;\ j \neq k, \\
& u_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\
& v_{ik} \geqslant 0, \quad i = 1, \ldots, m,
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\text{Maximize} \quad & \sum_{r=1}^{s} u_{rk}\left(\sum_{j=1, j\neq k}^{n} y_{rj}\right) \\
\text{Subject to} \quad & \sum_{i=1}^{m} v_{ik}\left(\sum_{j=1, j\neq k}^{n} x_{ij}\right) = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^{m} v_i x_{ik} = 0, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leqslant 0, \quad j = 1, \ldots, n;\ j \neq k, \\
& u_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\
& v_{ik} \geqslant 0, \quad i = 1, \ldots, m.
\end{aligned} \tag{5}$$

Model (4) is known as the aggressive formulation for cross-efficiency evaluation which aims to minimize the cross-efficiencies of the other DMUs in some way, whereas model (5) is known as the benevolent formulation for cross-efficiency evaluation which maximizes the cross-efficiencies of the other DMUs to some extent. Since the two models optimize the input and output weights in two different ways, there is thus no guarantee that they can lead to the same efficiency ranking or conclusion for the $n$ DMUs.

Other alternative secondary goals or models suggested in the DEA literature include:

- Minimize $\sum_{r=1}^{s} u_{rk}\left(\sum_{j=1, j\neq k}^{n} y_{rj}\right) - \sum_{i=1}^{m} v_{ik}\left(\sum_{j=1, j\neq k}^{n} x_{ij}\right)$ (Sexton et al., 1986).
- Replace the constraint $\sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^{m} v_i x_{ik} = 0$ in model (4) by $\sum_{r=1}^{s} u_{rk} y_{rk} - \alpha_k \sum_{i=1}^{m} v_i x_{ik} \geqslant 0$, where $\alpha_k \in (\min_{1\leqslant j\leqslant n} \theta_{jj}, 1)$ is a parameter set to allow model (4) to maximize the cross-efficiencies of the other $(n-1)$ DMUs by holding the efficiency of DMU$_k$ no less than a given parameter value (Wu, 2009).
- MinMaximize $\alpha_j'$ or Minimize $\frac{1}{n}\sum_{j=1}^{n} \left|\alpha_j' - \bar{\alpha}'\right|$ subject to $\sum_{i=1}^{m} v_{ik} x_{ik} = 1,\ \theta_{kk}^* = \sum_{r=1}^{s} u_{rk} y_{rk}$ and $\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \alpha_j' = 0$ for $j = 1, \ldots, n$, where $\bar{\alpha}\prime = \frac{1}{n}\sum_{j=1}^{n} \alpha_j'$ (Liang et al., 2008b).

**Table 1**
Cross-efficiency matrix for $n$ DMUs.

| DMU | Target DMU | | | | Average cross-efficiency |
|---|---|---|---|---|---|
| | 1 | 2 | $\cdots$ | $n$ | |
| 1 | $\theta_{11}$ | $\theta_{12}$ | $\cdots$ | $\theta_{1n}$ | $\frac{1}{n}\sum_{k=1}^{n}\theta_{1k}$ |
| 2 | $\theta_{21}$ | $\theta_{22}$ | $\cdots$ | $\theta_{2n}$ | $\frac{1}{n}\sum_{k=1}^{n}\theta_{2k}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $n$ | $\theta_{n1}$ | $\theta_{n2}$ | $\cdots$ | $\theta_{nn}$ | $\frac{1}{n}\sum_{k=1}^{n}\theta_{nk}$ |

The above alternative goals or models are either aggressive or benevolent. In the next section, we will develop a neutral DEA model for cross-efficiency evaluation, which is neither aggressive nor benevolent. Each DMU determines the weights only from its own point of view without considering their impacts on the other DMUs. This good feature enables the decision maker (DM) not to make a difficult choice between the aggressive and benevolent formulations. The cross-efficiencies determined by the neutral DEA model will be more neutral just as its name implies.

### 3. A neutral DEA model for cross-efficiency evaluation

In our point of view, when a DMU is given an opportunity to decide a set of input and output weights, what the DMU is concerned most about is whether the weights can be as favorable as possible to itself. It should not care too much about how to be aggressive or benevolent to the other DMUs. Based upon this point of view, we construct the following neutral DEA model for the cross-efficiency evaluation of DMU$_k$:

$$\begin{aligned}
\text{Maximize} \quad & \delta = \underset{r\in\{1,\ldots,s\}}{\text{Minimum}} \left\{ \frac{u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \right\} \\
\text{Subject to} \quad & \theta_{kk}^* = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}, \\
\theta_{jk} = & \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \leqslant 1, \quad j = 1, \ldots, n;\ j \neq k, \\
& u_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\
& v_{ik} \geqslant 0, \quad i = 1, \ldots, m,
\end{aligned} \tag{6}$$

where $u_{rk} y_{rk} / \sum_{i=1}^{m} v_{ik} x_{ik}$ is the efficiency of the r$^{th}$ output of DMU$_k$. The economic meaning of the above model can be interpreted as "DMU$_k$ searches for a set of input and output weights to maximize its efficiency as a whole and at the same time to make its each output being as efficient as possible to produce sufficient efficiency as an individual".

Obviously, the goal of the above model has nothing to do with the other DMUs. It determines the input and output weights just from the angle of DMU$_k$ itself. The DM has therefore not to make any difficult yet subjective choice between the aggressive and benevolent formulations. This is the biggest advantage of the neutral DEA model over other DEA models for cross-efficiency evaluation. Through Charnes and Cooper transformation, model (6) can be converted into the following LP for solution:

$$\begin{aligned}
\text{Maximize} \quad & \delta \\
\text{Subject to} \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = \theta_{kk}^*, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leqslant 0, \quad j = 1, \ldots, n;\ j \neq k, \\
& u_{rk} y_{rk} - \delta \geqslant 0, \quad r = 1, \ldots, s, \\
& v_{ik} \geqslant 0, \quad i = 1, \ldots, m, \\
& \delta \geqslant 0,
\end{aligned} \tag{7}$$

where $u_{rk}$ $(r = 1, \ldots, s)$, $v_{ik}$ $(i = 1, \ldots, m)$ and $\delta$ are decision variables. Like all DEA models for cross-efficiency evaluation, model (7) needs to be solved $n$ times, each time for one DMU. Accordingly, there will be $n$ sets of input and output weights available for cross-efficiency evaluation.

Another significant advantage of the neutral DEA model over existing models for cross-efficiency evaluation is that the neutral DEA model can effectively reduce the number of zero weights for outputs. In other words, outputs in the neutral DEA models can be made as much use of as possible.

## 4. Extension to cross-weight evaluation

As is seen from model (7), each DMU determines one set of input and output weights for cross-efficiency evaluation. Accordingly, there are $n$ sets of input and output weights for $n$ DMUs. The cross-efficiency evaluation is to use these $n$ sets of weights to calculate $n$ efficiency values for each DMU and then average them into an overall efficiency score. This is only one of the possible ways for assessing and ranking DMUs. If the $n$ sets of weights are not directly used for computing the $n$ efficiency values of each DMU, but are used to generate an average set of input and output weights for the $n$ DMUs, then we get another way for assessing and ranking DMUs, which we refer to as cross-weight evaluation, as shown in Table 2.

In order to generate an average set of input and output weights for the $n$ DMUs, the $n$ sets of weights in Table 2, which we refer to as cross-weights, have to be comparable; otherwise, they cannot be averaged. It is easy to find that the input and output weights derived from model (7) are not comparable among DMUs because they meet the condition of $\sum_{i=1}^{m} v_{ik}x_{ik} = 1$, whereas $x_{ik}$ $(i = 1, \ldots, m)$ vary from one DMU to another. To make the weights derived from model (7) comparable among different DMUs, they need to be rescaled or normalized to meet the same standard. In this paper, we normalize the weights derived from model (7) to the same standard: $\sum_{i=1}^{m} v_{ik}\left(\sum_{j=1}^{n} x_{ij}\right) \equiv 1$. The following theorem shows how the normalization can be conducted to meet the set standard. Since its proof is trivial, we give the theorem without proof.

**Theorem 1.** *Denote by* $v_{ik}^*$ $(i = 1, \ldots, m)$ and $u_{rk}^*$ $(r = 1, \ldots, s)$ *the most favorable input and output weights for* $DMU_k$ *derived from model (7). Let*

$$\tilde{v}_{ik}^* = \frac{v_{ik}^*}{\sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)}, \quad i = 1, \ldots, m, \tag{8}$$

$$\tilde{u}_{rk}^* = \frac{u_{rk}^*}{\sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)}, \quad r = 1, \ldots, s. \tag{9}$$

*Then,* $\tilde{v}_{ik}^*$ $(i = 1, \ldots, m)$ and $\tilde{u}_{rk}^*$ $(r = 1, \ldots, s)$ *meet the condition of* $\sum_{i=1}^{m} \tilde{v}_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right) \equiv 1$.

It is easy to verify that the above normalized input and output weights $\tilde{v}_{ik}^*$ $(i = 1, \ldots, m)$ and $\tilde{u}_{rk}^*$ $(r = 1, \ldots, s)$ are the optimal solution to the following fractional program:

$$\begin{aligned} \text{Maximize} \quad & \delta = \underset{r\in\{1,\ldots,s\}}{\text{Minimum}} \left\{ \frac{\tilde{u}_{rk} y_{rk}}{\sum_{i=1}^{m} \tilde{v}_{ik} x_{ik}} \right\} \\ \text{Subject to} \quad & \sum_{i=1}^{m} \tilde{v}_{ik}\left(\sum_{j=1}^{n} x_{ij}\right) = 1, \\ & \theta_{kk}^* = \frac{\sum_{r=1}^{s} \tilde{u}_{rk} y_{rk}}{\sum_{i=1}^{m} \tilde{v}_{ik} x_{ik}}, \\ & \theta_{jk} = \frac{\sum_{r=1}^{s} \tilde{u}_{rk} y_{rj}}{\sum_{i=1}^{m} \tilde{v}_{ik} x_{ij}} \leqslant 1, \quad j = 1, \ldots, n;\ j \neq k, \\ & \tilde{u}_{rk} \geqslant 0, \quad r = 1, \ldots, s, \\ & \tilde{v}_{ik} \geqslant 0, \quad i = 1, \ldots, m, \end{aligned} \tag{10}$$

which is an extension of model (6) to make input and output weights comparable among DMUs.

After the $n$ sets of normalized input and output weights are obtained, the average cross-weights for the $n$ DMUs are generated as

$$\bar{v}_i^* = \frac{1}{n}\sum_{k=1}^{n} \tilde{v}_{ik}^*, \quad i = 1, \ldots, m, \tag{11}$$

$$\bar{u}_r^* = \frac{1}{n}\sum_{k=1}^{n} \tilde{u}_{rk}^*, \quad r = 1, \ldots, s, \tag{12}$$

which prove to be also normalized and meet the condition of $\sum_{i=1}^{m} \bar{v}_i^*\left(\sum_{j=1}^{m} x_{ij}\right) = 1$. This average set of cross-weights is then utilized to assess the performances of the $n$ DMUs. Let

$$\bar{\theta}_j = \frac{\sum_{r=1}^{s} \bar{u}_r^* y_{rj}}{\sum_{i=1}^{m} \bar{v}_i^* x_{ij}}, \quad j = 1, \ldots, n \tag{13}$$

be the efficiencies of the $n$ DMUs computed by using the average set of cross-weights. Then, we have

$$\begin{aligned} \bar{\theta}_j &= \frac{\sum_{r=1}^{s} \bar{u}_r^* y_{rj}}{\sum_{i=1}^{m} \bar{v}_i^* x_{ij}} = \frac{\sum_{r=1}^{s} \left(\frac{1}{n}\sum_{k=1}^{n} \tilde{u}_{rk}^*\right) y_{rj}}{\sum_{i=1}^{m} \left(\frac{1}{n}\sum_{k=1}^{n} \tilde{v}_{ik}^*\right) x_{ij}} = \frac{\sum_{k=1}^{n} \left(\sum_{r=1}^{s} \tilde{u}_{rk}^* y_{rj}\right)}{\sum_{k=1}^{n} \left(\sum_{i=1}^{m} \tilde{v}_{ik}^* x_{ij}\right)} \\ &= \frac{\sum_{k=1}^{n} \left(\sum_{r=1}^{s} u_{rk}^* y_{rj} \Big/ \sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)\right)}{\sum_{k=1}^{n} \left(\sum_{i=1}^{m} v_{ik}^* x_{ij} \Big/ \sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)\right)} \\ &= \frac{\sum_{k=1}^{n} \left(\theta_{jk} \sum_{i=1}^{m} v_{ik}^* x_{ij} \Big/ \sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)\right)}{\sum_{k=1}^{n} \left(\sum_{i=1}^{m} v_{ik}^* x_{ij} \Big/ \sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)\right)} \\ &= \sum_{k=1}^{n} \left( \frac{\sum_{i=1}^{m} v_{ik}^* x_{ij} \Big/ \sum_{i=1}^{m} v_{ik}^*\left(\sum_{j=1}^{n} x_{ij}\right)}{\sum_{l=1}^{n} \left(\sum_{i=1}^{m} v_{il}^* x_{ij} \Big/ \sum_{i=1}^{m} v_{il}^*\left(\sum_{j=1}^{n} x_{ij}\right)\right)} \right) \theta_{jk}, \end{aligned}$$

from which it is observed that $\bar{\theta}_j$ is a weighted average of the cross-efficiencies of $DMU_j$ and the weighting coefficients depend on the

**Table 2**
Cross-weight evaluation for $n$ DMUs.

| DMU | Input weights | | | | Output weights | | | |
|---|---|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $\cdots$ | $v_m$ | $u_1$ | $u_2$ | $\cdots$ | $u_s$ |
| 1 | $v_{11}$ | $v_{21}$ | $\cdots$ | $v_{m1}$ | $u_{11}$ | $u_{21}$ | $\cdots$ | $u_{s1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $k$ | $v_{1k}$ | $v_{2k}$ | $\cdots$ | $v_{mk}$ | $u_{1k}$ | $u_{2k}$ | $\cdots$ | $u_{sk}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $n$ | $v_{1n}$ | $v_{2n}$ | $\cdots$ | $v_{mn}$ | $u_{1n}$ | $u_{2n}$ | $\cdots$ | $u_{sn}$ |
| Average | $\frac{1}{n}\sum_{k=1}^{n} v_{1k}$ | $\frac{1}{n}\sum_{k=1}^{n} v_{2k}$ | $\cdots$ | $\frac{1}{n}\sum_{k=1}^{n} v_{mk}$ | $\frac{1}{n}\sum_{k=1}^{n} u_{1k}$ | $\frac{1}{n}\sum_{k=1}^{n} u_{2k}$ | $\cdots$ | $\frac{1}{n}\sum_{k=1}^{n} u_{sk}$ |

proportions of the total weighted input $\sum_{i=1}^{m} v_{ik}^{*} x_{ij}$ of DMU$_j$ to the total weighted input $\sum_{i=1}^{m} v_{ik}^{*} \left(\sum_{j=1}^{n} x_{ij}\right)$ of the $n$ DMUs under different cross-weights. Apparently, the weighting coefficients vary from DMU$_j$ to another.

Similar to the maverick index in the cross-efficiency evaluation suggested by Doyle and Green (1994) and the false positive index (FPI) defined by Baker and Talluri (1997), the following efficiency disparity index (EDI) can also be used for measuring the false positiveness of the DMUs:

$$EDI_k = \frac{\theta_{kk}^{*} - \bar{\theta}_k}{\bar{\theta}_k} \times 100\%, \quad k = 1, \ldots, n, \tag{14}$$

where $\theta_{kk}^{*}$ is the CCR-efficiency of DMU$_k$ and $\bar{\theta}_k$ is its efficiency computed by using the average set of cross-weights.

## 5. Comparisons with DEA game cross-efficiency evaluation

According to Liang et al. (2008a), DEA game cross-efficiency of DMU$_k$ relative to DMU$_d$ is defined as

$$\theta_{kd} = \frac{\sum_{r=1}^{s} u_{rk}^{d*} y_{rj}}{\sum_{i=1}^{m} v_{ik}^{d*} x_{ik}}, \quad d = 1, \ldots, n, \tag{15}$$

where $u_{rk}^{d*}$ $(r = 1, \ldots, s)$ and $v_{ik}^{d*}$ $(i = 1, \ldots, m)$ are the optimal weights determined by the following parametric LP model with $\alpha_d \leqslant 1$ being a parameter:

$$\begin{aligned}
\text{Maximize} \quad & \theta_{kd} = \sum_{r=1}^{s} u_{rk}^{d} y_{rk} \\
\text{Subject to} \quad & \sum_{i=1}^{m} v_{ik}^{d} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk}^{d} y_{rj} - \sum_{i=1}^{m} v_{ik}^{d} x_{ij} \leqslant 0, \quad j = 1, \ldots, n, \\
& \alpha_d \times \sum_{i=1}^{m} v_{ik}^{d} x_{id} - \sum_{r=1}^{s} u_{rk}^{d} y_{rd} \leqslant 0, \\
& u_{rk}^{d} \geqslant 0, \quad r = 1, \ldots, s, \\
& v_{ik}^{d} \geqslant 0, \quad i = 1, \ldots, m.
\end{aligned} \tag{16}$$

The above model aims at maximizing the efficiency of DMU$_k$ under the condition that the efficiency of a given DMU$_d$, namely, $\sum_{r=1}^{s} u_{rk}^{d} y_{rd} / \sum_{i=1}^{m} v_{ik}^{d} x_{id}$, is not less than a given value $\alpha_d$. The model is solved $n$ times, each time for a different $d \in \{1, \ldots, n\}$. As a result, there will be $n$ sets of input and output weights for DMU$_k$, leading to $n$ game cross-efficiencies for the DMU. The average game cross-efficiency of DMU$_k$ is thus defined as

$$\alpha_k = \frac{1}{n} \sum_{d=1}^{n} \theta_{kd} = \frac{1}{n} \sum_{d=1}^{n} \sum_{r=1}^{s} u_{rk}^{d*} y_{rk}. \tag{17}$$

Since $\theta_{kd}$ varies with the parameter $\alpha_d$, $\alpha_k$ will be a function of $(\alpha_1, \ldots, \alpha_n)$. To numerically determine the value of $\alpha_k$, the following iterative algorithm was developed by Liang et al. (2008a):

*Step 1*. Solve model (3) and obtain a set of original average DEA cross-efficiency scores. Let $t = 1$ and $\alpha_d = \alpha_d^1 = \overline{E}_d$, where $\overline{E}_d$ is the average cross-efficiency score of DMU$_d$, $d = 1, \ldots, n$.

*Step 2*. Solve model (16). Let

$$\alpha_k^{t+1} = \frac{1}{n} \sum_{d=1}^{n} \sum_{r=1}^{s} u_{rk}^{d*}(\alpha_d^t) y_{rk}, \tag{18}$$

where $u_{rk}^{d*}(\alpha_d^t)$ represents the optimal value of $u_{rk}^{d}$ in model (16) when $\alpha_d = \alpha_d^t$.

*Step 3*. If $\left|\alpha_k^{t+1} - \alpha_k^t\right| \geqslant \varepsilon$ for some $k \in \{1, \ldots, n\}$, where $\varepsilon$ is a prespecified small positive value such as 0.001, then let $\alpha_d = \alpha_d^{t+1}$ and go to Step 2. If $\left|\alpha_k^{t+1} - \alpha_k^t\right| < \varepsilon$ for all $k = 1, \ldots, n$, then stop. $\alpha_k^{t+1}$ will be the best average game cross-efficiency of DMU$_k$.

It has been proved by Liang et al. (2008a) that the above iterative algorithm is convergent and the best average game cross-efficiency scores for the $n$ DMUs constitute a Nash equilibrium point. From the above introduction, we can find the differences between the conventional cross-efficiency and the game cross-efficiency, which are stated as follows:

- In the conventional cross-efficiency evaluation, each DMU determines one set of input and output weights for all the DMUs. As a result, the sets of input and output weights for computing cross-efficiencies are unified and the same for all the DMUs. In the DEA game cross-efficiency evaluation, however, each DMU determines $n$ sets of input and output weights for itself rather than for all the DMUs. Consequently, the sets of weights for calculating game cross-efficiencies may differ from one DMU to another.
- The sets of weights for computing cross-efficiencies in the conventional cross-efficiency evaluation are determined by different DMUs. They cannot be all favorable or unfavorable to a particular DMU. The sets of weights for calculating game cross-efficiencies, however, are all determined in a favorable way by the same DMU. In other words, they are all favorable to the DMU. This is the reason why DEA game cross-efficiencies are always higher than the conventional cross-efficiencies.
- In the conventional cross-efficiency evaluation, two LP models need to be solved for each DMU, one for determining its best CCR-efficiency and the other for determining a set of input and output weights for computing cross-efficiencies. As a result, there are $2n$ LP models to be solved for $n$ DMUs in total. In the DEA game cross-efficiency evaluation, however, one has to solve $n$ LP models for each DMU in every round of iteration. Accordingly, there are $n^2$ LP models in total to be solved for $n$ DMUs in each round of iteration. Although the iterative algorithm presented by Liang et al. proves to be convergent, the computational effort required by the DEA game cross-efficiency evaluation is obviously very demanding.
- DEA game cross-efficiencies prove to be unique and in particular, average DEA game cross-efficiencies constitute a Nash equilibrium point. The conventional cross-efficiencies, however, can usually be determined in different ways such as aggressive, benevolent or neutral. Once the way is selected, the cross-efficiencies also turn out to be unique. In particular, the neutral DEA model developed in Section 3 computes the cross-efficiencies in a neutral way, thus avoiding the difficulty in making a subjective choice between the benevolent and aggressive formulations.

## 6. Numerical examples

In this section, we provide two numerical examples to illustrate the inconsistency between the orderings achieved by the aggressive and benevolent formulations for cross-efficiency evaluation and the applications of the proposed neutral DEA model and the cross-weight evaluation in DEA ranking.

**Example 1** (Efficiency evaluation of seven departments in a university (Wong & Beasley, 1990)). Seven academic departments (DMUs) in a university are evaluated in terms of three inputs and three outputs given below and their input and output data are documented in Table 3.

**Table 3**
Data for seven academic departments in a university.

| Department (DMU) | Inputs | | | Outputs | | | CCR-efficiency |
|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $y_1$ | $y_2$ | $y_3$ | |
| 1 | 12 | 400 | 20 | 60 | 35 | 17 | 1 |
| 2 | 19 | 750 | 70 | 139 | 41 | 40 | 1 |
| 3 | 42 | 1500 | 70 | 225 | 68 | 75 | 1 |
| 4 | 15 | 600 | 100 | 90 | 12 | 17 | 0.8197 |
| 5 | 45 | 2000 | 250 | 253 | 145 | 130 | 1 |
| 6 | 19 | 730 | 50 | 132 | 45 | 45 | 1 |
| 7 | 41 | 2350 | 600 | 305 | 159 | 97 | 1 |

$x_1$: Number of academic staff.
$x_2$: Academic staff salaries in thousands of pounds.
$x_3$: Support staff salaries in thousands of pounds.
$y_1$: Number of undergraduate students.
$y_2$: Number of postgraduate students.
$y_3$: Number of research papers.

It is verified that output 3 ($y_3$) has no impact on the CCR-efficiencies of the seven academic departments and is therefore removed from the outputs for cross-evaluation. However, it may still have an effect on other efficiencies such as BCC or slack-based efficiencies. When these models are used, output 3 should not be dropped out. Since the CCR-efficiencies evaluate six of the seven academic departments as efficient and cannot discriminate among them any further, cross-efficiencies are computed. Tables 4 and 5 show the aggressive and benevolent cross-efficiencies of the seven academic departments, which are obtained through the solutions of models (4) and (5). It is seen very clearly that the two different formulations for cross-efficiency evaluation result in two different efficiency rankings for the seven academic departments. In particular, the aggressive cross-efficiencies in Table 4 evaluate $DMU_1$ as the most efficient academic department, whereas the benevolent cross-efficiencies in Table 5 rank it just in the third place. The two different efficiency rankings cause, to somewhat extent, damages to the cross-efficiency evaluation which claims to produce a unique ordering for DMUs.

To avoid the difficulty in making a subjective choice between the two different formulations for cross-efficiency evaluation, we use the proposed neutral DEA model (7) to re-evaluate the cross-efficiencies of the seven academic departments. The results are shown in Table 6. Since these cross-efficiencies are neither aggressive nor benevolent, they are more neutral. The average cross-efficiencies in Table 6 clearly show that $DMU_1$ is less efficient than $DMU_6$ but performs better than any other academic departments.

In Table 7 we show the cross-weight evaluation results for the seven academic departments, where the normalized input and output weights of the seven academic departments are obtained by normalizing the optimal solutions of model (7) in terms of Eqs. (8) and (9). The average set of cross-weights for the seven academic departments are provided in the last row of the table and are used to evaluate the efficiencies of the seven academic departments. The results are presented in the last but one column of

**Table 4**
Aggressive cross-efficiency matrix of the seven academic departments in a university.

| Department (DMU) | Target DMU | | | | | | | Average cross-efficiency | Ranking |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | |
| 1 | 1.0000 | 0.8452 | 0.9333 | 0.6878 | 1.0000 | 0.9333 | 0.7521 | 0.8788 | 1 |
| 2 | 0.3347 | 1.0000 | 0.6178 | 1.0000 | 0.7017 | 0.8426 | 0.5564 | 0.7219 | 4 |
| 3 | 0.5551 | 0.8481 | 1.0000 | 0.7351 | 0.5551 | 1.0000 | 0.4175 | 0.7301 | 3 |
| 4 | 0.0686 | 0.7551 | 0.2800 | 0.8197 | 0.2417 | 0.4413 | 0.2063 | 0.4018 | 7 |
| 5 | 0.3314 | 0.6620 | 0.3148 | 0.7646 | 1.0000 | 0.4778 | 0.8309 | 0.6259 | 5 |
| 6 | 0.5143 | 1.0000 | 0.8213 | 0.9507 | 0.7915 | 1.0000 | 0.6107 | 0.8126 | 2 |
| 7 | 0.1514 | 0.6044 | 0.1581 | 0.9985 | 0.9854 | 0.2783 | 1.0000 | 0.5966 | 6 |

**Table 5**
Benevolent cross-efficiency matrix of the seven academic departments in a university.

| Department (DMU) | Target DMU | | | | | | | Average cross-efficiency | Ranking |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | |
| 1 | 1.0000 | 0.9219 | 1.0000 | 0.6875 | 1.0000 | 1.0000 | 1.0000 | 0.9442 | 3 |
| 2 | 0.9812 | 1.0000 | 0.8510 | 1.0000 | 0.8461 | 0.9812 | 0.9812 | 0.9486 | 2 |
| 3 | 0.7690 | 0.7719 | 1.0000 | 0.7349 | 0.6651 | 0.7690 | 0.7690 | 0.7827 | 6 |
| 4 | 0.6411 | 0.7013 | 0.4542 | 0.8197 | 0.4135 | 0.6411 | 0.6411 | 0.6160 | 7 |
| 5 | 0.9382 | 0.8990 | 0.4950 | 0.7650 | 1.0000 | 0.9382 | 0.9382 | 0.8534 | 5 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 0.9506 | 0.9104 | 1.0000 | 1.0000 | 0.9801 | 1 |
| 7 | 1.0000 | 1.0000 | 0.2941 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.8992 | 4 |

**Table 6**
Cross-efficiency matrix of the seven academic departments in a university by the neutral DEA model.

| Department (DMU) | Target DMU | | | | | | | Average cross-efficiency | Ranking |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | |
| 1 | 1.0000 | 0.9219 | 1.0000 | 0.6874 | 1.0000 | 1.0000 | 0.9444 | 0.9362 | 2 |
| 2 | 0.9302 | 1.0000 | 0.6166 | 1.0000 | 0.8461 | 0.9538 | 0.9718 | 0.9026 | 3 |
| 3 | 0.7590 | 0.7719 | 1.0000 | 0.7349 | 0.6651 | 0.7491 | 0.7544 | 0.7763 | 6 |
| 4 | 0.6143 | 0.7013 | 0.2680 | 0.8197 | 0.4135 | 0.5409 | 0.5966 | 0.5649 | 7 |
| 5 | 0.8360 | 0.8990 | 0.3365 | 0.7649 | 1.0000 | 0.9960 | 0.9584 | 0.8272 | 5 |
| 6 | 0.9550 | 1.0000 | 0.8294 | 0.9506 | 0.9104 | 1.0000 | 1.0000 | 0.9493 | 1 |
| 7 | 0.8193 | 1.0000 | 0.1670 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.8552 | 4 |

**Table 7**
Cross-weight evaluation of the seven academic departments in a university.

| Department (DMU) | Normalized input and output weights | | | | | Efficiency by average cross-weights | Rank |
|---|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $u_1$ | $u_2$ | | |
| 1 | 0 | 1.20048E−04 | 0 | 4.00160E−04 | 6.85989E−04 | 0.9235 | 2 |
| 2 | 2.59619E−03 | 5.98961E−05 | 0 | 5.05473E−04 | 5.85097E−04 | 0.9185 | 3 |
| 3 | 0 | 0 | 8.62069E−04 | 2.47603E−04 | 6.81477E−05 | 0.7516 | 6 |
| 4 | 4.97096E−03 | 4.87449E−06 | 0 | 7.05785E−04 | 1.76205E−11 | 0.5586 | 7 |
| 5 | 4.40352E−03 | 0 | 1.29415E−04 | 1.96745E−04 | 1.24645E−03 | 0.8320 | 4 |
| 6 | 4.35477E−03 | 0 | 1.37525E−04 | 3.44397E−04 | 9.81255E−04 | 0.9591 | 1 |
| 7 | 4.47506E−03 | 0 | 1.17511E−04 | 4.16368E−04 | 7.98693E−04 | 0.7632 | 5 |
| Average | 2.97150E−03 | 2.64027E−05 | 1.78074E−04 | 4.02362E−04 | 6.23662E−04 | – | – |

**Table 8**
False positiveness of the seven academic departments in a university.

| Department (DMU) | FPI (%) for cross-efficiency evaluation | | | EDI (%) for cross-weight evaluation |
|---|---|---|---|---|
| | Aggressive | Benevolent | Neutral | |
| 1 | **13.79** | 5.91 | 6.81 | 8.29 |
| 2 | 38.52 | 5.41 | 10.79 | 8.87 |
| 3 | 36.96 | 27.76 | 28.81 | 33.05 |
| 4 | 104.01 | 33.07 | 45.11 | 46.74 |
| 5 | 59.76 | 17.18 | 20.88 | 20.19 |
| 6 | 23.06 | **2.03** | **5.34** | **4.26** |
| 7 | 67.62 | 11.21 | 16.94 | 31.03 |

**Table 9**
Data for 14 passenger airlines.

| Airline (DMU) | Inputs | | | Outputs | | CCR-efficiency |
|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $y_1$ | $y_2$ | |
| 1 | 5723 | 3239 | 2003 | 26,677 | 697 | 0.8684 |
| 2 | 5895 | 4225 | 4557 | 3081 | 539 | 0.3379 |
| 3 | 24,099 | 9560 | 6267 | 124,055 | 1266 | 0.9475 |
| 4 | 13,565 | 7499 | 3213 | 64,734 | 1563 | 0.9581 |
| 5 | 5183 | 1880 | 783 | 23,604 | 513 | 1.0000 |
| 6 | 19,080 | 8032 | 3272 | 95,011 | 572 | 0.9766 |
| 7 | 4603 | 3457 | 2360 | 22,112 | 969 | 1.0000 |
| 8 | 12,097 | 6779 | 6474 | 52,363 | 2001 | 0.8588 |
| 9 | 6587 | 3341 | 3581 | 26,504 | 1297 | 0.9477 |
| 10 | 5654 | 1878 | 1916 | 19,277 | 972 | 1.0000 |
| 11 | 12,559 | 8098 | 3310 | 41,925 | 3398 | 1.0000 |
| 12 | 5728 | 2481 | 2254 | 27,754 | 982 | 1.0000 |
| 13 | 4715 | 1792 | 2485 | 31,332 | 543 | 1.0000 |
| 14 | 22,793 | 9874 | 4145 | 122,528 | 1404 | 1.0000 |

Table 7, which gives similar ranking to that in Table 6. This shows the usefulness of the cross-weight evaluation in DEA ranking and discriminating between DEA efficient units.

To illustrate the application of the efficiency disparity index (EDI) defined by Eq. (14) in measuring the false positiveness of DMUs, we show in Table 8 the values of FPI and EDI of the seven academic departments, where FPI was computed by (Baker & Talluri, 1997)

$$\mathrm{FPI}_k = \frac{\theta_{kk}^* - \left(\frac{1}{n}\sum_{j=1}^{n}\theta_{kj}\right)}{\left(\frac{1}{n}\sum_{j=1}^{n}\theta_{kj}\right)} \times 100\%, \quad k = 1, \ldots, n.$$

It is seen that none of the six DEA efficient academic departments is highly false positive. However, the non-DEA efficient academic department $DMU_4$ exhibits a high false positiveness. This shows that there is a significant difference between the self-evaluation and the peer-evaluation of $DMU_4$. Usually, a lower value of FPI or

**Table 10**
Aggressive cross-efficiency matrix of the 14 passenger airlines.

| Airline (DMU) | Target DMU | | | | | | | | | | | | | | Average cross-efficiency | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | | |
| 1 | 0.8684 | 0.4501 | 0.6225 | 0.8684 | 0.4418 | 0.4726 | 0.7679 | 0.7881 | 0.7031 | 0.4158 | 0.3390 | 0.7043 | 0.4711 | 0.4726 | 0.5990 | 12 |
| 2 | 0.1719 | 0.3379 | 0.0472 | 0.1719 | 0.0224 | 0.0247 | 0.2770 | 0.2724 | 0.2808 | 0.2465 | 0.1152 | 0.2789 | 0.0417 | 0.0247 | 0.1652 | 14 |
| 3 | 0.8826 | 0.1942 | 0.9475 | 0.8826 | 0.6567 | 0.6898 | 0.6468 | 0.6833 | 0.6225 | 0.2559 | 0.1968 | 0.6261 | 0.7422 | 0.6898 | 0.6226 | 11 |
| 4 | 0.9581 | 0.4259 | 0.7034 | 0.9581 | 0.6683 | 0.6973 | 0.7629 | 0.7850 | 0.6991 | 0.4027 | 0.4739 | 0.7016 | 0.4937 | 0.6973 | 0.6734 | 7 |
| 5 | 0.9653 | 0.3658 | 1.0000 | 0.9653 | 1.0000 | 1.0000 | 0.7011 | 0.7359 | 0.7778 | 0.5272 | 0.6382 | 0.7820 | 0.7181 | 1.0000 | 0.7983 | 1 |
| 6 | 0.8818 | 0.1108 | 0.9563 | 0.8818 | 0.9632 | 0.9766 | 0.5745 | 0.6084 | 0.5099 | 0.1376 | 0.1703 | 0.5141 | 0.6766 | 0.9766 | 0.6385 | 9 |
| 7 | 0.9211 | 0.7781 | 0.4773 | 0.9211 | 0.3108 | 0.3382 | 1.0000 | 1.0000 | 0.8395 | 0.5416 | 0.4000 | 0.8383 | 0.3658 | 0.3382 | 0.6478 | 8 |
| 8 | 0.7813 | 0.6114 | 0.5162 | 0.7813 | 0.2683 | 0.2924 | 0.8415 | 0.8588 | 0.8208 | 0.5703 | 0.3011 | 0.8194 | 0.4418 | 0.2924 | 0.5855 | 13 |
| 9 | 0.7855 | 0.7278 | 0.5076 | 0.7855 | 0.2455 | 0.2677 | 0.8881 | 0.9072 | 0.9477 | 0.7501 | 0.3528 | 0.9452 | 0.4537 | 0.2677 | 0.6309 | 10 |
| 10 | 0.7821 | 0.6354 | 0.6520 | 0.7821 | 0.3338 | 0.3564 | 0.7650 | 0.7944 | 1.0000 | 1.0000 | 0.4942 | 1.0000 | 0.5871 | 0.3564 | 0.6813 | 6 |
| 11 | 1.0000 | 1.0000 | 0.4287 | 1.0000 | 0.4202 | 0.4418 | 1.0000 | 1.0000 | 1.0000 | 0.8107 | 1.0000 | 1.0000 | 0.2961 | 0.4418 | 0.7742 | 2 |
| 12 | 0.9462 | 0.6336 | 0.7500 | 0.9462 | 0.4085 | 0.4395 | 0.9082 | 0.9395 | 0.9998 | 0.7647 | 0.4244 | 1.0000 | 0.6398 | 0.4395 | 0.7314 | 5 |
| 13 | 1.0000 | 0.4257 | 1.0000 | 1.0000 | 0.4183 | 0.4555 | 0.9511 | 1.0000 | 1.0000 | 0.5855 | 0.2129 | 1.0000 | 1.0000 | 0.4555 | 0.7503 | 3 |
| 14 | 1.0000 | 0.2277 | 1.0000 | 1.0000 | 0.9806 | 1.0000 | 0.6919 | 0.7275 | 0.6478 | 0.2747 | 0.3300 | 0.6521 | 0.7097 | 1.0000 | 0.7316 | 4 |

EDI is more preferred. The most efficient DMU is undoubtedly the one with the lowest FPI or EDI value, as highlighted in Table 8.

**Example 2** (Efficiency evaluation of 14 international passenger airlines (Tofallis, 1997)). Fourteen major international passenger airlines are evaluated in terms of three inputs and two outputs that are defined below:

$x_1$: aircraft capacity in ton kilometres.
$x_2$: operating cost.
$x_3$: non-flight assets such as reservation systems, facilities, and current assets.
$y_1$: passenger kilometres.
$y_2$: non-passenger revenue.

**Table 11**
Benevolent cross-efficiency matrix of the 14 passenger airlines.

| Airline (DMU) | Target DMU | | | | | | | | | | | | | | Average cross-efficiency | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | | |
| 1 | 0.8684 | 0.4501 | 0.6225 | 0.8684 | 0.8492 | 0.4726 | 0.8108 | 0.7881 | 0.7031 | 0.7512 | 0.8684 | 0.7713 | 0.8684 | 0.8684 | 0.7543 | 12 |
| 2 | 0.1719 | 0.3379 | 0.0472 | 0.1719 | 0.1735 | 0.0247 | 0.2479 | 0.2724 | 0.2808 | 0.2058 | 0.1719 | 0.2025 | 0.1719 | 0.1719 | 0.1894 | 14 |
| 3 | 0.8826 | 0.1942 | 0.9475 | 0.8826 | 0.8844 | 0.6898 | 0.7232 | 0.6833 | 0.6225 | 0.7846 | 0.8826 | 0.8072 | 0.8826 | 0.8826 | 0.7678 | 9 |
| 4 | 0.9581 | 0.4259 | 0.7034 | 0.9581 | 0.9413 | 0.6973 | 0.8228 | 0.7850 | 0.6991 | 0.8113 | 0.9581 | 0.8341 | 0.9581 | 0.9581 | 0.8222 | 6 |
| 5 | 0.9653 | 0.3658 | 1.0000 | 0.9653 | 1.0000 | 1.0000 | 0.7704 | 0.7359 | 0.7778 | 1.0000 | 0.9653 | 1.0000 | 0.9653 | 0.9653 | 0.8912 | 3 |
| 6 | 0.8818 | 0.1108 | 0.9563 | 0.8818 | 0.8780 | 0.9766 | 0.6615 | 0.6084 | 0.5099 | 0.7176 | 0.8818 | 0.7478 | 0.8818 | 0.8818 | 0.7554 | 11 |
| 7 | 0.9211 | 0.7781 | 0.4773 | 0.9211 | 0.8795 | 0.3382 | 1.0000 | 1.0000 | 0.8395 | 0.7808 | 0.9211 | 0.8012 | 0.9211 | 0.9211 | 0.8214 | 7 |
| 8 | 0.7813 | 0.6114 | 0.5162 | 0.7813 | 0.7703 | 0.2924 | 0.8458 | 0.8588 | 0.8208 | 0.7532 | 0.7813 | 0.7631 | 0.7813 | 0.7813 | 0.7242 | 13 |
| 9 | 0.7855 | 0.7278 | 0.5076 | 0.7855 | 0.7889 | 0.2677 | 0.8782 | 0.9072 | 0.9477 | 0.8375 | 0.7855 | 0.8369 | 0.7855 | 0.7855 | 0.7590 | 10 |
| 10 | 0.7821 | 0.6354 | 0.6520 | 0.7821 | 0.8250 | 0.3564 | 0.7780 | 0.7944 | 1.0000 | 1.0000 | 0.7821 | 0.9719 | 0.7821 | 0.7821 | 0.7803 | 8 |
| 11 | 1.0000 | 1.0000 | 0.4287 | 1.0000 | 1.0000 | 0.4418 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9193 | 1 |
| 12 | 0.9462 | 0.6336 | 0.7500 | 0.9462 | 0.9602 | 0.4395 | 0.9362 | 0.9395 | 0.9998 | 1.0000 | 0.9462 | 1.0000 | 0.9462 | 0.9462 | 0.8850 | 4 |
| 13 | 1.0000 | 0.4257 | 1.0000 | 1.0000 | 1.0000 | 0.4555 | 1.0000 | 1.0000 | 1.0000 | 0.9843 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9190 | 2 |
| 14 | 1.0000 | 0.2277 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.7795 | 0.7275 | 0.6478 | 0.8569 | 1.0000 | 0.8838 | 1.0000 | 1.0000 | 0.8659 | 5 |

**Table 12**
Cross-efficiency matrix of the 14 passenger airlines by the neutral DEA model.

| Airline (DMU) | Target DMU | | | | | | | | | | | | | | Average cross-efficiency | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | | |
| 1 | 0.8684 | 0.4501 | 0.6225 | 0.8684 | 0.4907 | 0.4726 | 0.7744 | 0.7881 | 0.7031 | 0.6603 | 0.8639 | 0.7533 | 0.7031 | 0.8492 | 0.7049 | 11 |
| 2 | 0.1719 | 0.3379 | 0.0472 | 0.1719 | 0.1027 | 0.0247 | 0.2716 | 0.2724 | 0.2808 | 0.1641 | 0.1723 | 0.2054 | 0.2808 | 0.1735 | 0.1912 | 14 |
| 3 | 0.8826 | 0.1942 | 0.9475 | 0.8826 | 0.4856 | 0.6898 | 0.6606 | 0.6833 | 0.6225 | 0.7894 | 0.8830 | 0.7871 | 0.6225 | 0.8844 | 0.7154 | 10 |
| 4 | 0.9581 | 0.4259 | 0.7034 | 0.9581 | 0.7093 | 0.6973 | 0.7704 | 0.7850 | 0.6991 | 0.7109 | 0.9542 | 0.8137 | 0.6991 | 0.9413 | 0.7733 | 7 |
| 5 | 0.9653 | 0.3658 | 1.0000 | 0.9653 | 1.0000 | 1.0000 | 0.7091 | 0.7359 | 0.7778 | 1.0000 | 0.9730 | 1.0000 | 0.7778 | 1.0000 | 0.8764 | 2 |
| 6 | 0.8818 | 0.1108 | 0.9563 | 0.8818 | 0.6162 | 0.9766 | 0.5895 | 0.6084 | 0.5099 | 0.7125 | 0.8809 | 0.7209 | 0.5099 | 0.8780 | 0.7024 | 12 |
| 7 | 0.9211 | 0.7781 | 0.4773 | 0.9211 | 0.4738 | 0.3382 | 1.0000 | 1.0000 | 0.8395 | 0.6340 | 0.9110 | 0.7829 | 0.8395 | 0.8795 | 0.7711 | 8 |
| 8 | 0.7813 | 0.6114 | 0.5162 | 0.7813 | 0.3741 | 0.2924 | 0.8434 | 0.8588 | 0.8208 | 0.6645 | 0.7787 | 0.7542 | 0.8208 | 0.7703 | 0.6906 | 13 |
| 9 | 0.7855 | 0.7278 | 0.5076 | 0.7855 | 0.4036 | 0.2677 | 0.8865 | 0.9072 | 0.9477 | 0.7504 | 0.7863 | 0.8374 | 0.9477 | 0.7889 | 0.7378 | 9 |
| 10 | 0.7821 | 0.6354 | 0.6520 | 0.7821 | 0.5587 | 0.3564 | 0.7632 | 0.7944 | 1.0000 | 1.0000 | 0.7918 | 0.9969 | 1.0000 | 0.8250 | 0.7813 | 6 |
| 11 | 1.0000 | 1.0000 | 0.4287 | 1.0000 | 1.0000 | 0.4418 | 0.9901 | 1.0000 | 1.0000 | 0.7970 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9041 | 1 |
| 12 | 0.9462 | 0.6336 | 0.7500 | 0.9462 | 0.5418 | 0.4395 | 0.9115 | 0.9395 | 0.9998 | 0.9399 | 0.9494 | 1.0000 | 0.9998 | 0.9602 | 0.8541 | 4 |
| 13 | 1.0000 | 0.4257 | 1.0000 | 1.0000 | 0.3800 | 0.4555 | 0.9652 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9860 | 1.0000 | 1.0000 | 0.8723 | 3 |
| 14 | 1.0000 | 0.2277 | 1.0000 | 1.0000 | 0.7507 | 1.0000 | 0.7058 | 0.7275 | 0.6478 | 0.8286 | 1.0000 | 0.8598 | 0.6478 | 1.0000 | 0.8140 | 5 |

**Table 13**
Cross-weight evaluation of the 14 passenger airlines.

| Airline (DMU) | Normalized input–output weights | | | | | Efficiency by average cross-weights | Rank |
|---|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $u_1$ | $u_2$ | | |
| 1 | 5.03141E−06 | 0 | 5.44695E−06 | 9.85695E−07 | 1.17403E−05 | 0.7021 | 11 |
| 2 | 6.74395E−06 | 0 | 0 | 1.09403E−12 | 2.49256E−05 | 0.1698 | 14 |
| 3 | 3.73965E−07 | 9.68589E−06 | 5.27362E−06 | 1.02851E−06 | 1.02692E−12 | 0.7117 | 9 |
| 4 | 5.03141E−06 | 6.90892E−14 | 5.44695E−06 | 9.85695E−07 | 1.17403E−05 | 0.7710 | 6 |
| 5 | 4.95101E−08 | 0 | 2.12925E−05 | 3.58598E−07 | 1.64997E−05 | 0.8518 | 2 |
| 6 | 3.95132E−07 | 0 | 2.01933E−05 | 7.56621E−07 | 7.39793E−13 | 0.6849 | 12 |
| 7 | 6.74395E−06 | 0 | 0 | 7.01936E−07 | 1.60178E−05 | 0.7424 | 8 |
| 8 | 6.42868E−06 | 6.48085E−07 | 0 | 7.19866E−07 | 1.64231E−05 | 0.6652 | 13 |
| 9 | 3.29439E−06 | 7.09094E−06 | 0 | 5.05526E−07 | 2.28377E−05 | 0.7056 | 10 |
| 10 | 0 | 1.13035E−05 | 3.96017E−06 | 6.80796E−07 | 1.61440E−05 | 0.7671 | 7 |
| 11 | 5.13373E−06 | 0 | 5.12151E−06 | 9.71100E−07 | 1.19816E−05 | 0.9023 | 1 |
| 12 | 1.83136E−06 | 7.09852E−06 | 4.64161E−06 | 6.94741E−07 | 1.96353E−05 | 0.8420 | 3 |
| 13 | 3.29439E−06 | 7.09094E−06 | 0 | 5.05526E−07 | 2.28377E−05 | 0.8378 | 4 |
| 14 | 4.16379E−06 | 1.58310E−06 | 5.75703E−06 | 9.46934E−07 | 1.30867E−05 | 0.8001 | 5 |
| Average | 9.84155E−06 | 2.03870E−04 | 4.85157E−05 | 4.45009E−05 | 7.71337E−05 | – | – |

**Table 14**
False positiveness of the 14 passenger airlines.

| Airline (DMU) | FPI (%) for cross-efficiency evaluation | | | EDI (%) for cross-weight evaluation |
|---|---|---|---|---|
| | Aggressive | Benevolent | Neutral | |
| 1 | 44.98 | 15.12 | 26.10 | 23.67 |
| 2 | 104.52 | 78.39 | 111.58 | 98.97 |
| 3 | 52.18 | 23.40 | 40.38 | 33.13 |
| 4 | 42.28 | 16.53 | 26.30 | 24.27 |
| 5 | **25.26** | 12.21 | 17.49 | 17.40 |
| 6 | 52.96 | 29.28 | 53.16 | 42.59 |
| 7 | 54.36 | 21.74 | 35.55 | 34.69 |
| 8 | 46.67 | 18.59 | 29.76 | 29.09 |
| 9 | 50.23 | 24.86 | 35.18 | 34.33 |
| 10 | 46.77 | 28.16 | 32.27 | 30.37 |
| 11 | 29.16 | **8.78** | **14.54** | **10.82** |
| 12 | 36.72 | 13.00 | 20.40 | 18.76 |
| 13 | 33.28 | 8.82 | 20.90 | 19.36 |
| 14 | 36.69 | 15.48 | 29.70 | 24.99 |

Table 9 shows the input and output data of the 14 passenger airlines in the year 1990 together with their CCR-efficiencies, which evaluate seven out of the 14 passenger airlines as DEA efficient and cannot distinguish them any further.

Tables 10 and 11 show, respectively, the aggressive and benevolent cross-efficiencies of the 14 passenger airlines. It is found that the two different formulations for cross-efficiency evaluation result in two different efficiency rankings again. According to the aggressive cross-efficiencies in Table 10, $DMU_5$ is the most efficient passenger airline. However, the benevolent cross-efficiencies in Table 11 evaluate $DMU_{11}$ as the most efficient passenger airline. Such an inconsistency in efficiency ranking obviously causes confusion for the DM. It is very natural that the DM would like to know which conclusion or ranking is more credible. Such a question cannot be answered by any of existing DEA cross-efficiency models.

In comparison with the aggressive and benevolent cross-efficiencies in Tables 10 and 11, the cross-efficiencies presented in Table 12, which are obtained by solving the neutral DEA model (7), are more neutral, neither aggressive nor benevolent. According to the neutral cross-efficiencies in Table 12, $DMU_{11}$ is the most efficient passenger airline, whereas $DMU_5$ is the second most efficient, followed by $DMU_{13}$, $DMU_{12}$, $DMU_{14}$, and others.

We also conduct a cross-weight evaluation for the 14 passenger airlines. The results are provided in Table 13. The efficiencies computed by using the average set of cross-weights also evaluate $DMU_{11}$ as the most efficient passenger airline, followed by $DMU_5$, $DMU_{12}$, $DMU_{13}$, $DMU_{14}$, and others.

Table 14 shows the values of FPI and EDI of the 14 passenger airlines, from which it is seen that none of the seven DEA efficient passenger airlines is highly false positive, but a non-DEA efficient passenger airline $DMU_2$ reveals a very high false positiveness. This implies that the self-evaluation of $DMU_2$ is significantly different from its peer-evaluation. The most efficient passenger airline is the DMU with the lowest FPI or EDI value, as highlighted in Table 14. It shows again the inconsistency between the aggressive and benevolent cross-efficiency evaluations.

## 7. Conclusions

Cross-efficiency evaluation is an important method for comparing and ranking DMUs. Existing DEA models for cross-efficiency evaluation are either aggressive or benevolent. It has been shown that the two different formulations, aggressive and benevolent, may result in two distinct efficiency rankings or conclusions. This causes damages to the use of cross-efficiency evaluation which has long been recommended and claimed to be able to produce a unique ordering for DMUs. Moreover, different formulations for cross-efficiency evaluation also cause difficulty for the DM to make a subjective choice between the two formulations. As a matter of fact, there is no evidence to support the DM to choose either of them.

To resolve these problems, we have proposed in this paper a neutral DEA model for cross-efficiency evaluation. The neutral DEA model does not require the DM to make a subjective choice between the two different formulations. Each DMU determines the weights purely from its own point of view without the need to be aggressive or benevolent to the other DMUs. As a result, the cross-efficiencies computed in this way are more neutral and more logical. We have also extended the neutral DEA model and proposed a cross-weight evaluation, which seeks a common set of weights for all the DMUs. We have shown how the weights derived for cross-efficiency evaluation can be normalized for cross-weight evaluation. Its usefulness has been illustrated with two numerical examples.

In conclusion, we summarize below the good and unique features that the neutral DEA model has:

- Each DMU determines input and output weights from its own point of view without the need of being aggressive or benevolent to any other DMUs.
- The weights and cross-efficiencies determined by the neutral DEA model are neutral, neither aggressive nor benevolent.
- Outputs can be made as much use of as possible and the number of zero weights for outputs can be significantly reduced.
- The neutral DEA model is much easier to solve than DEA game cross-efficiency model.

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