J. Opl Res. Soc. Vol. 41, No. 9, pp. 829–835, 1990
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0160-5682/90 $3.50 + 0.00

Restricting Weight Flexibility in Data Envelopment Analysis

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In this paper we present a method, based on the use of proportions, for restricting weight flexibility in data envelopment analysis. This method is applicable when the decision-making units being evaluated have multiple inputs and outputs.

Key words: data envelopment analysis, efficiency measurement, weights restrictions

INTRODUCTION

In this paper we present a method for restricting weight flexibility in data envelopment analysis (DEA). As far as we are aware, this method has not been presented before in the literature. We shall assume throughout this paper some familiarity on the part of the reader with DEA.

BASIC DEA MODEL

The basic DEA model is as follows. Let

- s be the number of output measures;
- t be the number of input measures;
- n be the number of decision-making units (DMUs) which are being evaluated with respect to one another;
- y_{ik} be the value ($\geqslant 0$) of output measure i ($i = 1, \ldots, s$) for DMU k ($k = 1, \ldots, n$);
- x_{jk} be the value ($\geqslant 0$) of input measure j ($j = 1, \ldots, t$) for DMU k ($k = 1, \ldots, n$);
- u_i be the weight ($\geqslant 0$) to be attached to output measure i ($i = 1, \ldots, s$);
- v_j be the weight ($\geqslant 0$) to be attached to input measure j ($j = 1, \ldots, t$);
- e_k be the (relative) efficiency of DMU k ($k = 1, \ldots, n$);
- ε be a very small 'non-Archimedean' number (>0).

Conceptually DEA defines a total output S_k for each DMU k as a weighted sum of outputs; i.e.

$$S_k = \sum_{i=1}^{s} u_i y_{ik} \qquad k = 1, \ldots, n, \tag{1}$$

where the units of measurement for the weights (u_i) are such that the product ($u_i y_{ik}$) is dimensionless. In this way outputs (y_{ik}) whose units of measurement are different can be combined into a total output S_k, which is also dimensionless. The total input T_k is defined in a similar way; i.e.

$$T_k = \sum_{j=1}^{t} v_j x_{jk} \qquad k = 1, \ldots, n, \tag{2}$$

so that it is also dimensionless.

We determine the efficiency (e_K) of DMU K using the non-linear programme $P(e_K)$ given by

$$\text{maximize} \quad e_K \tag{3}$$

$$\text{subject to} \quad e_k = \left(\sum_{i=1}^{s} u_i y_{ik}\right) \Big/ \left(\sum_{j=1}^{t} v_j x_{jk}\right) \qquad k = 1, \ldots, n \tag{4}$$

$$e_k \leqslant 1 \qquad k = 1, \ldots, n \tag{5}$$

$$u_i \geqslant \varepsilon \qquad i = 1, \ldots, s \tag{6}$$

$$v_j \geqslant \varepsilon \qquad j = 1, \ldots, t \tag{7}$$

Equation (4) defines efficiency e_k as total output (S_k) divided by total input (T_k). Equation (5) is taken from an engineering analogy[1,2] so that $e_k \leqslant 1$ implies that total output $\leqslant$ total input (i.e. $S_k \leqslant T_k$). Equations (6) and (7) ensure that all weights are non-zero. This non-linear programme can be converted into a linear programme using an approach due to Charnes and Cooper[1,3,4] and hence can be easily solved.

DEA was first proposed by Charnes *et al.*[1] in 1978. Since then the method has been applied in a number of areas, notably in education,[5–17] but also in health care,[18–25] the US armed forces,[26–30] local government,[31–33] the police,[24,34] banking,[35,36] law courts,[37] prisons,[38] electric utilities,[39] fast food restaurants[40] and tax collection.[41]

A considerable amount of (essentially) theoretical work relating both to DEA[18,22,40,42–55] and to efficiency[56–62] has also appeared in the literature. A recent bibliography by Seiford[63] contains 400 references and is indicative of the amount of interest that DEA has attracted.

The role of ε has been the subject of some discussion in the literature.[62,64–66] In practice ε is taken to be a very small number,[22,26,40,65] and its purpose is basically twofold:

(a) mathematical—to ensure that the denominator of the right-hand side of equation (4) is never zero (assuming that $\sum_{j=1}^{t} x_{jk} > 0$, $k = 1, \ldots, n$, i.e. that all DMUs have at least one non-zero input measure); and
(b) conceptual—to ensure that all input/output measures are assigned some weight (however small).

EXAMPLE

In order to illustrate and motivate the method we have developed for restricting weight flexibility we shall consider a small example. The original impetus for the work presented in this paper arose out of a study concerned with tax collection.[41] However, subsequently one of the authors[17] has applied the method presented in this paper in the context of a study comparing university departments, and since this is an area that should be more familiar to the majority of readers, our example is drawn from it.

Consider Table 1, where we present hypothetical data for seven university departments with three input measures (number of academic staff, academic staff salaries, support staff salaries) and three output measures (number of undergraduate students, number of postgraduate students, number of research papers).

TABLE 1. *Data*

	Input measure			Output measure		
Department (DMU)	1 Number of academic staff	2 Academic staff salaries (£'000)	3 Support staff salaries (£'000)	1 Number of undergraduate students	2 Number of postgraduate students	3 Number of research papers
1	12	400	20	60	35	17
2	19	750	70	139	41	40
3	42	1500	70	225	68	75
4	15	600	100	90	12	17
5	45	2000	250	253	145	130
6	19	730	50	132	45	45
7	41	2350	600	305	159	97

Applying the basic DEA model given before to this data, we obtain the results (to three decimal places) shown in Table 2, where we have taken ε to be 10^{-6} and have scaled the weights after solution such that the first output weight greater than ε has value 100.

Considering Table 2, it is clear that, for any DMU, the weights used in evaluating the efficiency of that DMU are such that some input/output measures are (effectively) ignored. This is plainly unsatisfactory, and it is to help overcome this that we have developed the method presented below for restricting weight flexibility.

TABLE 2. *Basic DEA results*

Department (DMU)	Efficiency	Input weights v_1	v_2	v_3	Output weights u_1	u_2	u_3
1	1	266	0.440	6.600	0	100	0
2	1	727.433	0	1.125	100	0	0
3	1	0.001	8.410	141.214	100	0	0
4	0.820	704.316	0.691	0	100	0	0
5	1	0.001	11.583	0	0	100	66.667
6	1	1290.571	0	40.818	100	291.969	4.959
7	1	266	0.440	6.600	0	100	0

Note: A weight of zero in Tables 2 and 3 implies a weight less than 0.001.

Note here that it is not possible to conclude from Table 2 that for a particular DMU a particular input/output measure must be ignored in order for the DMU to achieve maximum efficiency (as there may be alternative optimal solutions to the basic DEA model for the DMU which lead to the same maximum efficiency).

There have been only a few papers presented in the literature concerned with restricting weight flexibility in DEA. Dyson and Thanassoulis[33] presented an approach valid in the case of just a single input measure. Charnes *et al.*[35,49] and Thompson *et al.*[67,68] have presented an approach based upon imposing limits on ratios of weights.

RESTRICTING WEIGHT FLEXIBILITY

The method we have developed for restricting weight flexibility in DEA is based upon the use of *proportions*. As mentioned above, S_k is a dimensionless measure of the total output of DMU k. Since, by definition, $\sum_{i=1}^{s} (u_i y_{ik}/S_k) = 1$, we have that $u_i y_{ik}/S_k$ represents the proportion of the total output for DMU k devoted to output measure i. Conceptually we can regard $u_i y_{ik}/S_k$ as representing the 'importance' attached to output measure i by DMU k since the larger this expression, the more DMU k depends upon output measure i in determining its efficiency.

Now it may be that, from consideration of the particular problem being modelled via DEA, the decision-maker (or modeller) can set limits $[a_i, b_i]$ $(0 \leqslant a_i \leqslant b_i \leqslant 1)$ on what is regarded as suitable lower and upper limits for the importance of output measure i in DMU k; i.e. they wish to have

$$a_i \leqslant u_i y_{ik}/S_k \leqslant b_i. \tag{8}$$

Specification of $[a_i, b_i]$ is a *value judgement*. Such judgements mean that the situation being modelled is, in the view of the decision-maker, better represented by imposing such limits.

Usually such limits are arrived at by seeking a consensus amongst those familiar with the situation being modelled as to the relative importance of each output measure in the total output. Obtaining such limits is not too difficult. Questions of the form

(a) 'Do you think that the importance of output measure i in evaluating DMUs could be as low (as high) as $z\%$?'; or
(b) 'Should, as a matter of policy, the importance of output measure i in evaluating DMUs be allowed to be as low (as high) as $z\%$?'

(for varying values of z) can be used to elicit such limits.

It may be, of course, that no consensus can be reached. In that case no limits can be imposed (as in the basic DEA model—equations (3)–(7)). However, we still end up with a model which is at least as good as the basic DEA model (if no consensus can be arrived at) and maybe better (if a consensus on limits has been arrived at).

Note here that although we have only discussed proportion constraints for the output measures above, it is clear that the same basic approach can also be applied to input measures.

Note also that after adding proportion constraints to the basic DEA model, the resulting non-linear programme can still be converted into a linear programme.

There are a number of approaches to using the proportion constraint (equation (8)):

(a) Add equation (8) to $P(e_K)$ for $k = K$; i.e. the weights chosen to evaluate DMU K (maximize e_k) must be such that for DMU K (but not necessarily for any other DMU) the proportion constraint is satisfied.

The logic here is that if the proportion constraint is to be satisfied it should at least be satisfied by the DMU (K) being evaluated.

(b) Add equation (8) to $P(e_K)$ for $k = 1, 2, \ldots, K, \ldots, n$; i.e. the weights chosen to evaluate DMU K (maximize e_K) must be such that for *all* DMUs the proportion constraint is satisfied.

The logic behind this approach is modelled on that used in basic DEA. In basic DEA (equations (3)–(7) above) the weights chosen to evaluate DMU K must be such that its efficiency is $\leqslant 1$. When the *same* set of weights is applied to all other DMUs, their efficiencies must also be $\leqslant 1$.

Applying exactly the same logic to the proportion constraint implies that the weights chosen to evaluate DMU K must be such that its proportion constraint is satisfied and when the *same* set of weights is applied to all other DMUs, their proportion constraints must also be satisfied.

The difficulty with this approach is computational. Each output (or input) measure for which a proportion constraint is applied adds $2n$ inequality constraints to the non-linear programme $P(e_K)$ and hence also adds $2n$ inequality constraints to the linear programme derived from $P(e_K)$. If several proportion constraints are to be applied and n is large, then this approach becomes expensive computationally (in the work reported in Wong[41], for example, n was 332).

(c) As approach (a) above but also add to $P(e_K)$ the constraint

$$a_i \leqslant \left[u_i\left(\sum_{k=1}^{n} y_{ik}/n\right)\right] \Big/ \left[\sum_{j=1}^{s} u_j\left(\sum_{k=1}^{n} y_{jk}/n\right)\right] \leqslant b_i, \tag{9}$$

which represents the proportion constraint for the 'average' DMU, i.e. the proportion constraint for an artificial DMU with the value of output measure i for this artificial DMU being

$$\left(\sum_{k=1}^{n} y_{ik}/n\right),$$

the average of the values for output measure i over all n DMUs.

The logic behind this approach is to try and overcome the computational difficulties associated with approach (b) above by applying the proportion constraint both to the DMU (K) being evaluated and to the 'average' DMU.

For computational reasons, we generally use approach (c) above.

EXAMPLE

In order to illustrate proportion constraints, consider the small example presented above concerned with university departments (Tables 1 and 2). We shall discuss just two proportion constraints, one concerned with an output measure and the other concerned with an input measure. We consider each in turn.

For DMU k in Table 1 the proportion of total output associated with the measure of research output (number of research papers) is given by

$$(u_3 y_{3k})/(\textstyle\sum_{i=1}^{3} u_i y_{ik}).$$

General expectations about what constitutes a university department lead us to believe that research output should be an important component of total departmental output. Hence we could reasonably expect that this should be reflected in the value of

$$(u_3 y_{3k})/(\textstyle\sum_{i=1}^{3} u_i y_{ik}).$$

However, the results in Table 2 for the basic DEA model imply values of 0, 0, 0, 0, 0.374, 0.017 and 0 for this proportion for the seven DMUs, indicating that five of the seven DMUs give zero weight to research output. This is plainly unsatisfactory. Whilst different people will naturally have different ideas (different value judgements) on the importance of research output in a university department, for the purposes of illustration, we use

$$0.3 \leqslant (u_3 y_{3k}) \Big/ \left(\sum_{i=1}^{3} u_i y_{ik} \right) \leqslant 0.6; \tag{10}$$

i.e. the proportion of total output concerned with research output should lie between 0.3 and 0.6.

Turning now to input measures, we have that for DMU k in Table 1 the proportion of total input associated with support staff salaries is given by

$$(v_3 x_{3k})/(\textstyle\sum_{j=1}^{3} v_j x_{jk}).$$

The results in Table 2 for the basic DEA model imply values of 0.038, 0.006, 0.439, 0, 0, 0.077 and 0.249 for this proportion for the seven DMUs. Considering Table 1, it is clear that for DMUs 3, 4 and 5 in particular, these proportions (0.439, 0 and 0 respectively) are unrealistic. In order to perturb the results for these DMUs we apply the proportion constraint

$$0.006 \leqslant (v_3 x_{3k}) \Big/ \left(\sum_{j=1}^{3} v_j x_{jk} \right) \leqslant 0.249. \tag{11}$$

Note here that this illustrates that proportion constraints can be derived by considering the proportions given by the results of the basic DEA model.

The effect of adding proportion constraints (10) and (11) (using approach (c) above) to the basic DEA model is shown in Table 3.

TABLE 3. *DEA results with proportions*

Department (DMU)	Efficiency	Input weights			Output weights		
		v_1	v_2	v_3	u_1	u_2	u_3
1	1	0	89.249	50.458	100	562.749	647.805
2	0.995	985.534	1.495	1.711	100	0	148.929
3	0.862	0.001	22.361	53.240	100	0	128.571
4	0.691	1160.381	1.782	1.243	100	0	226.891
5	1	1791.522	24.445	157.030	100	602.048	432.067
6	1	981.772	0	4.069	100	0	125.714
7	1	1047.092	9.916	1.481	100	103.688	207.598

Comparing Table 2 with Table 3, it is clear that there are now substantially fewer instances where an input/output measure is (effectively) ignored in calculating the efficiency for a DMU. We could, of course, continue to add any proportion constraints we consider valid if we wished.

Note here that using proportion constraints can result in it being impossible to calculate an efficiency for some DMUs (because their non-linear programmes are infeasible when the constraints associated with proportions are added). This can be for two possible reasons:

(a) some DMUs have an unusual profile of values for input/output measures and need closer attention; and/or

(b) there is insufficient flexibility associated with one (or more) of the proportion constraints (e.g. consider the effect of allowing no flexibility in the proportion constraints so that they are all equality constraints).

Discrimination between these two cases can be provided by applying proportion constraints using approaches (a) or (c) above. If infeasibility is detected for a small number of DMUs, then it indicates that reason (a) above applies to these DMUs. If infeasibility is detected for a large number of DMUs, then it indicates that reason (b) above applies.

CONCLUSIONS

In this paper we have illustrated the use of proportions in restricting weight flexibility in data envelopment analysis. We hope it is clear that, whatever the exact nature of the situation being considered, it may be possible to use proportions to restrict weight flexibility and hence obtain a better model for evaluating the efficiencies of the decision-making units.

In conclusion we would comment that we believe that restricting weight flexibility in data envelopment analysis is important and is an area in which further research is needed.

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