MAJORIZATION METHODS FOR MULTIVARIATE BEHRENS-FISHER PROBLEMS

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ABSTRACT. Meet the abstract. This is the abstract.

1. NEGATIVE LOG LIKELIHOOD

Suppose $\underline{y}_i$ are n independent multivariate normal random vectors, with $\mathbf{E}(\underline{y}_i) = \mu$ for all i. We suppose the index set $\mathscr{I} = 1, 2, \cdots, n$ is partitioned into m groups $\mathscr{I}_1, \cdots, \mathscr{I}_m$, with $n_1, ..., n_m$ elements. Moreover $\mathbf{V}(\underline{y}_i) = \Sigma_j$ for all $i \in \mathscr{I}_j$.

Twice the negative log likelihood is clearly

$$\mathscr{D} = \sum_{j=1}^{m} n_j \log \mathbf{det}(\Sigma_j) + \sum_{i \in \mathscr{I}_j} (\underline{y}_j - \mu)' \Sigma_j^{-1} (\underline{y}_j - \mu))$$

2. CONCENTRATING THE LIKELIHOOD

It follows that the maximum likelihood estimate of Σ_j, for given μ, is

$$\hat{\Sigma}_j = \frac{1}{n_j} \sum_{i \in \mathscr{I}} (y_i - \mu)(y_i - \mu)' = S_j + (\bar{y}_j - \mu)(\bar{y}_j - \mu)',$$

where $\bar{y}_j$ is the mean of group j and $S_j = \frac{1}{n_j} \sum_{i \in \mathscr{I}} (y_i - \bar{y}_j)(y_i - \bar{y}_j)'$ is the dispersion matrix.

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We concentrate the likelihood by substituting the values of S_j, so that the resulting concentrated likelihood is now only a function of μ. It is

$$\mathscr{D}_\star(\mu)=\sum_{j=1}^m n_j \log \mathbf{det}(S_j+(\overline{y}_j-\mu)(\overline{y}_j-\mu)')$$

3. MAJORIZATION

We now use the classical result of Ky Fan that $\log\mathbf{det}(X)$ is concave in X. Beckenbach and Bellman [1965, Chapter 2, Paragraph 9] or Magnus and Neudecker [1998, Chapter 11, Section 22].Because a concave function is below its tangents, we have for all X and $\tilde{X}$ that

$$\log\mathbf{det}(X)\leq\log\mathbf{det}(\tilde{X})+\mathbf{tr}\,\tilde{X}^{-1}(X-\tilde{X})$$

And thus the majorization function is

$$\mathscr{D}_\star(\mu)\leq\mathscr{D}_\star(\tilde{\mu})+\sum_{j=1}^m n_j\Sigma_j^{-1}(\tilde{\mu})(S_j+(\overline{y}_j-\mu)(\overline{y}_j-\mu)')$$

and this results in the majorization algorithm

$$\mu^{(k+1)}=(\sum_{j=1}^m n_j\Sigma_j(\mu^{(k)}))^{-1}(\sum_{j=1}^m n_j\Sigma_j(\mu^{(k)})\overline{y}_j.$$

Thus we update by computing a matrix weighted average [Chamberlain and Leamer, 1976] of the m means.

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